Example 3. To graduate from Simplexville University, Angie needs to pass at least one of the three courses she is taking this semester: literature, finance, and statistics. Angie's busy schedule of extracurricular activities allows her to spend only 4 hours per week on studying. Angie's probability of passing each course depends on the number of hours she spends studying for the course:

	Probability of passing course		
Hours of studying per week	Literature	Finance	Statistics
0	0.20	0.25	0.10
1	0.30	0.30	0.30
2	0.35	0.33	0.40
3	0.38	0.35	0.44
4	0.40	0.38	0.50

Angie wants to maximize the probability that she passes at least one of these three courses. Formulate this problem as a dynamic program by giving its shortest/longest path representation.

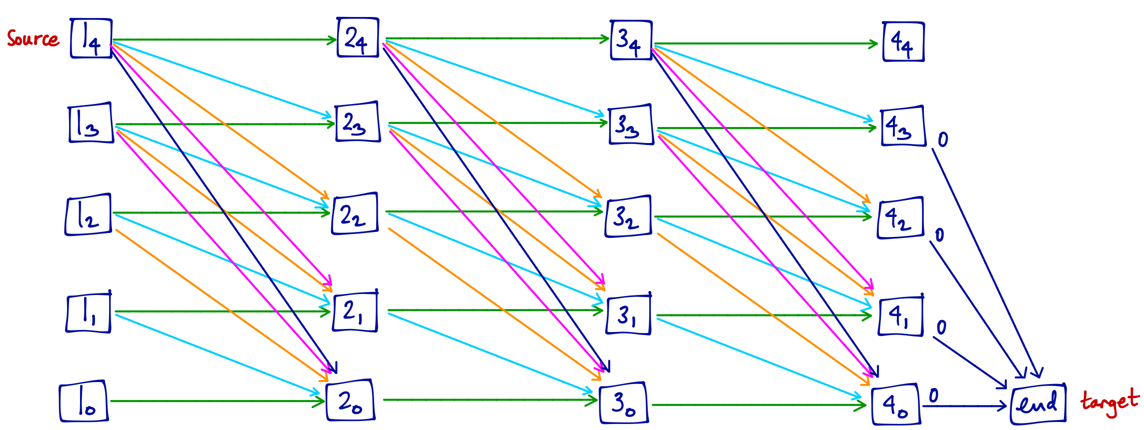
Hint. Why is maximizing the probability of passing at least one course equivalent to minimizing the probability of failing all three courses?

Note that $\Pr\{\text{passing at least 1 course}\} = 1 - \Pr\{\text{failing all 3 courses}\}$

Stage t: how many hours to devote to course t?

Node t_n: n hours left at stage t (with courses $t, t+1, \dots$ remaining)

Find a shortest path from source to target:



Shortest path length ≈ -1.23

$\Rightarrow e^{-1.23} = 0.293 =$ minimum possible prob. of failing all 3 courses

Shortest path: $1_4 \to 2_2 \to 3_2 \to 4_0$

$\Rightarrow$ 2 hours to literature
2 hours to statistics

Course	Hours	Edges	Edge lengths
Lit	0	$(1_n, 2_n)$ for $n = 0,1,2,3,4$	$\log(0.80)$
Lit	1	$(1_n, 2_{n-1})$ for $n = 1,2,3,4$	$\log(0.70)$
Lit	2	$(1_n, 2_{n-2})$ for $n = 2,3,4$	$\log(0.65)$
Lit	3	$(1_n, 2_{n-3})$ for $n = 3,4$	$\log(0.62)$
Lit	4	$(1_n, 2_{n-4})$ for $n = 4$	$\log(0.60)$
Fin	0	$(2_n, 3_n)$ for $n = 0,1,2,3,4$	$\log(0.75)$
Fin	1	$(2_n, 3_{n-1})$ for $n = 1,2,3,4$	$\log(0.70)$
Fin	2	$(2_n, 3_{n-2})$ for $n = 2,3,4$	$\log(0.67)$
Fin	3	$(2_n, 3_{n-3})$ for $n = 3,4$	$\log(0.65)$
Fin	4	$(2_n, 3_{n-4})$ for $n = 4$	$\log(0.62)$

Course	Hours	Edges	Edge lengths
Stat	0	$(3_n, 4_n)$ for $n = 0,1,2,3,4$	$\log(0.90)$
Stat	1	$(3_n, 4_{n-1})$ for $n = 1,2,3,4$	$\log(0.70)$
Stat	2	$(3_n, 4_{n-2})$ for $n = 2,3,4$	$\log(0.60)$
Stat	3	$(3_n, 4_{n-3})$ for $n = 3,4$	$\log(0.56)$
Stat	4	$(3_n, 4_{n-4})$ for $n = 4$	$\log(0.50)$